

Exercise 13

Find the limit.

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 - 9}}{2x - 6}$$

Solution

Plugging in ∞ right away for x gives ∞/∞ , which is meaningless. Simplify the function first or rewrite the limit.

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 - 9}}{2x - 6} &= \lim_{x \rightarrow \infty} \frac{\sqrt{(x+3)(x-3)}}{2(x-3)} \\ &= \lim_{x \rightarrow \infty} \frac{1}{2} \sqrt{\frac{x+3}{x-3}} \end{aligned}$$

Make the substitution, $u = x - 3$. Then as $x \rightarrow \infty$, so does u .

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 - 9}}{2x - 6} &= \lim_{u \rightarrow \infty} \frac{1}{2} \sqrt{\frac{(u+3)+3}{u}} \\ &= \lim_{u \rightarrow \infty} \frac{1}{2} \sqrt{\frac{u+6}{u}} \\ &= \lim_{u \rightarrow \infty} \frac{1}{2} \sqrt{\frac{u}{u} + \frac{6}{u}} \\ &= \lim_{u \rightarrow \infty} \frac{1}{2} \sqrt{1 + \frac{6}{u}} \\ &= \frac{1}{2} \sqrt{1 + 0} \\ &= \frac{1}{2} \end{aligned}$$